Introduction to Fracture Mechanics

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Introduction

In 1983, the National Bureau of Standards (now the National Institute for Science and Technology) and Battelle Memorial Institute[1] estimated the costs for failure due to fracture to be $119 billion per year in 1982 dollars. The dollars are important, but the cost of many failures in human life and injury is infinitely more so.

Failures have occurred for many reasons, including uncertainties in the loading or environment, defects in the materials, inadequacies in design, and deficiencies in construction or maintenance. Design against fracture has a technology of its own, and this is a very active area of current research. This module will provide an introduction to an important aspect of this field, since without an understanding of fracture the methods in stress analysis discussed previously would be of little use. We will focus on fractures due to simple tensile overstress, but the designer is cautioned again about the need to consider absolutely as many factors as possible that might lead to failure, especially when life is at risk.

The Module on the Dislocation Basis of Yield (Module 21) shows how the strength of structural metals – particularly steel – can be increased to very high levels by manipulating the microstructure so as to inhibit dislocation motion. Unfortunately, this renders the material increasingly brittle, so that cracks can form and propagate catastrophically with very little warning. An unfortunate number of engineering disasters are related directly to this phenomenon, and engineers involved in structural design must be aware of the procedures now available to safeguard against brittle fracture.

The central difficulty in designing against fracture in high-strength materials is that the presence of cracks can modify the local stresses to such an extent that the elastic stress analyses done so carefully by the designers are insufficient. When a crack reaches a certain critical length, it can propagate catastrophically through the structure, *even though the gross stress is much less than would normally cause yield or failure in a tensile specimen.* The term "fracture mechanics" refers to a vital specialization within solid mechanics in which the presence of a crack is assumed, and we wish to find quantitative relations between the crack length, the material's inherent resistance to crack growth, and the stress at which the crack propagates at high speed to cause structural failure.

[1] R.P. Reed et al., NBS Special Publication 647-1, Washington, 1983.

The energy-balance approach

When A.A. Griffith (1893–1963) began his pioneering studies of fracture in glass in the years just prior to 1920, he was aware of Inglis' work in calculating the stress concentrations around elliptical holes[2], and naturally considered how it might be used in developing a fundamental approach to predicting fracture strengths. However, the Inglis solution poses a mathematical difficulty: in the limit of a perfectly sharp crack, the stresses approach infinity at the crack tip. This is obviously nonphysical (actually the material generally undergoes some local yielding to blunt the cracktip), and using such a result would predict that materials would have near-*zero* strength: even for very small applied loads, the stresses near crack tips would become infinite, and the bonds there would rupture. Rather than focusing on the crack-tip stresses directly, Griffith employed an energy-balance approach that has become one of the most famous developments in materials science[3].

The strain energy per unit volume of stressed material is

$$U^* = \frac{1}{V}\int f\,dx = \int \frac{f}{A}\frac{dx}{L} = \int \sigma d\epsilon$$

If the material is linear ($\sigma = E\epsilon$), then the strain energy per unit volume is

$$U^* = \frac{E\epsilon^2}{2} = \frac{\sigma^2}{2E}$$

When a crack has grown into a solid to a depth a, a region of material adjacent to the free surfaces is unloaded, and its strain energy released. Using the Inglis solution, Griffith was able to compute just how much energy this is.

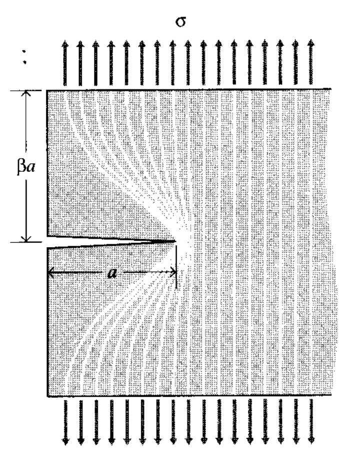

Figure 1: Idealization of unloaded region near crack flanks.

A simple way of visualizing this energy release, illustrated in Fig. 1, is to regard two triangular regions near the crack flanks, of width a and height βa, as being completely unloaded, while the remaining material continues to feel the full stress σ. The parameter β can be selected so as to

[2]See Module 16.

[3]A.A. Griffith, *Philosophical Transactions,* Series A, Vol. 221, pp. 163–198, 1920. The importance of Griffith's work in fracture was largely unrecognized until the 1950's. See J.E. Gordon, *The Science of Structures and Materials,* Scientific American Library, 1988, for a personal account of the Griffith story.

agree with the Inglis solution, and it turns out that for plane stress loading $\beta = \pi$. The total strain energy U released is then the strain energy per unit volume times the volume in both triangular regions:

$$U = -\frac{\sigma^2}{2E} \cdot \pi a^2$$

Here the dimension normal to the x-y plane is taken to be unity, so U is the strain energy released per unit thickness of specimen. This strain energy is *liberated* by crack growth. But in forming the crack, bonds must be broken, and the requisite bond energy is in effect *absorbed* by the material. The surface energy S associated with a crack of length a (and unit depth) is:

$$S = 2\gamma a$$

where γ is the surface energy (e.g., Joules/meter^2) and the factor 2 is needed since two free surfaces have been formed. As shown in Fig. 2, the total energy associated with the crack is then the sum of the (positive) energy absorbed to create the new surfaces, plus the (negative) strain energy liberated by allowing the regions near the crack flanks to become unloaded.

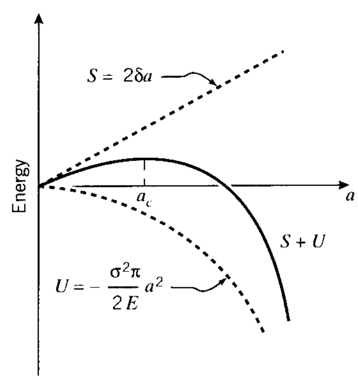

Figure 2: The fracture energy balance.

As the crack grows longer (a increases), the quadratic dependence of strain energy on a eventually dominates the surface energy, and beyond a critical crack length a_c the system can lower its energy by letting the crack grow still longer. Up to the point where $a = a_c$, the crack will grow only if the stress in increased. Beyond that point, crack growth is spontaneous and catastrophic.

The value of the critical crack length can be found by setting the derivative of the total energy $S + U$ to zero:

$$\frac{\partial(S+U)}{\partial a} = 2\gamma - \frac{\sigma_f^2}{E}\pi a = 0$$

Since fast fracture is imminent when this condition is satisfied, we write the stress as σ_f. Solving,

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}$$

Griffith's original work dealt with very brittle materials, specifically glass rods. When the material exhibits more ductility, consideration of the surface energy alone fails to provide an accurate model for fracture. This deficiency was later remedied, at least in part, independently by Irwin[4] and Orowan[5]. They suggested that in a ductile material a good deal – in fact the vast majority – of the released strain energy was absorbed not by creating new surfaces, but by energy dissipation due to plastic flow in the material near the crack tip. They suggested that catastrophic fracture occurs when the strain energy is released at a rate sufficient to satisfy the needs of all these energy "sinks," and denoted this *critical strain energy release rate* by the parameter $\mathcal{G}_c$; the Griffith equation can then be rewritten in the form:

$$\boxed{\sigma_f = \sqrt{\frac{E\mathcal{G}_c}{\pi a}}} \tag{1}$$

This expression describes, in a very succinct way, the interrelation between three important aspects of the fracture process: the *material*, as evidenced in the critical strain energy release rate $\mathcal{G}_c$; the *stress level* σ_f; and the *size*, a, of the flaw. In a design situation, one might choose a value of a based on the smallest crack that could be easily detected. Then for a given material with its associated value of $\mathcal{G}_c$, the safe level of stress σ_f could be determined. The structure would then be sized so as to keep the working stress comfortably below this critical value.

Example 1

The story of the DeHavilland Comet aircraft of the early 1950's, in which at least two aircraft disintegrated in flight, provides a tragic but fascinating insight into the importance of fracture theory. It is an eerie story as well, having been all but predicted in a 1948 novel by Nevil Shute named *No Highway.* The book later became a movie starring James Stewart as a perserverant metallurgist convinced that his company's new aircraft (the "Reindeer") was fatally prone to metal fatigue. When just a few years later the Comet was determined to have almost exactly this problem, both the book and the movie became rather famous in the materials engineering community.

The postmortem study of the Comet's problems was one of the most extensive in engineering history[6]. It required salvaging almost the entire aircraft from scattered wreckage on the ocean floor and also involved full-scale pressurization of an aircraft in a giant water tank. Although valuable lessons were learned, it is hard to overstate the damage done to the DeHavilland Company and to the British aircraft industry in general. It is sometimes argued that the long predominance of the United States in commercial aircraft is due at least in part to the Comet's misfortune.

The Comet aircraft had a fuselage of clad aluminum, with $\mathcal{G}_c \approx 300$ in-psi. The hoop stress due to relative cabin pressurization was 20,000 psi, and at that stress the length of crack that will propagate catastrophically is

$$a = \frac{\mathcal{G}_c E}{\pi\sigma^2} = \frac{(300)(11 \times 10^6)}{\pi(20 \times 10^3)^2} = 2.62''$$

A crack would presumably be detected in routine inspection long before it could grow to this length. But in the case of the Comet, the cracks were propagating from rivet holes near the cabin windows. When the crack reached the window, the size of the window opening was effectively added to the crack length, leading to disaster.

Modern aircraft are built with this failure mode in mind, and have "tear strips" that are supposedly able to stop any rapidly growing crack. But this remedy is not always effective, as was demonstrated in 1988 when a B737 operated by Aloha Airlines had the roof of the first-class cabin tear away.. That aircraft had stress-corrosion damage at a number of rivets in the fuselage lap splices, and this permitted

[4]G.R. Irwin, "Fracture Dynamics," *Fracturing of Metals,* American Society for Metals, Cleveland, 1948.
[5]E. Orowan, "Fracture and Strength of Solids," Report of Progress in Physics, Vol. 12, 1949.
[6]T. Bishop, *Metal Progress,* Vol. 67, pp. 79–85, May 1955.

multiple small cracks to link up to form a large crack. A great deal of attention is currently being directed to protection against this sort of "multi-site damage."

It is important to realize that the critical crack length is an absolute number, not depending on the size of the structure containing it. Each time the crack jumps ahead, say by a small increment δa, an additional quantity of strain energy is released from the newly-unloaded material near the crack. Again using our simplistic picture of a triangular-shaped region that is at zero stress while the rest of the structure continues to feel the overall applied stress, it is easy to see in Fig. 3 that much more more energy is released due to the jump at position 2 than at position 1. This is yet another reason why small things tend to be stronger: they simply aren't large enough to contain a critical-length crack.

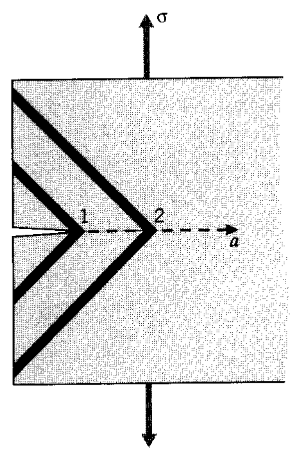

Figure 3: Energy released during an increment of crack growth, for two different crack lengths.

Example 2

Gordon[7] tells of a ship's cook who one day noticed a crack in the steel deck of his galley. His superiors assured him that it was nothing to worry about — the crack was certainly small compared with the vast bulk of the ship — but the cook began painting dates on the floor to mark the new length of the crack each time a bout of rough weather would cause it to grow longer. With each advance of the crack, additional decking material was unloaded, and the strain energy formerly contained in it released. But as the amount of energy released grows *quadratically* with the crack length, eventually enough was available to keep the crack growing even with no further increase in the gross load. When this happened, the ship broke into two pieces; this seems amazing but there are a more than a few such occurrences that are very well documented. As it happened, the part of the ship with the marks showing the crack's growth was salvaged, and this has become one of the very best documented examples of slow crack growth followed by final catastrophic fracture.

Compliance calibration

A number of means are available by which the material property $\mathcal{G}_c$ can be measured. One of these is known as *compliance calibration,* which employs the concept of compliance as a ratio of

[7] J.E. Gordon, *Structures, or Why Things Don't Fall Down,* Plenum, New York, 1978.